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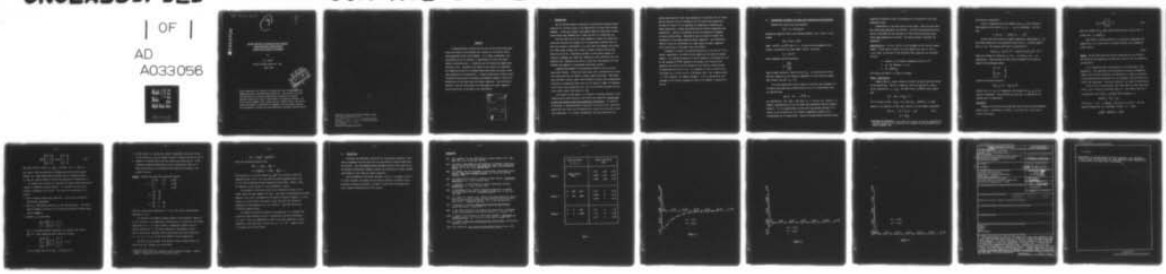

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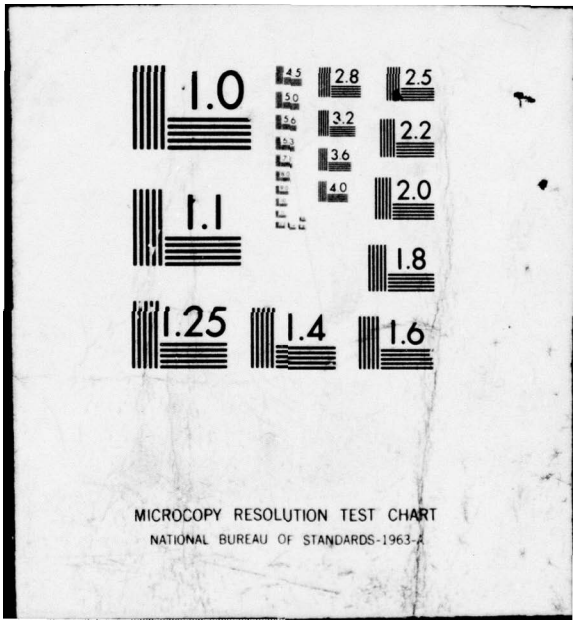
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ON THE FLEXIBILITY OFFERED BY STATE FEEDBACK IN MULTIVARIABLE SYSTEMS BEYOND CLOSED LOOP EIGENVALUE ASSIGNMENT

by

B. C. Moore*

Control Systems Report No. 7609

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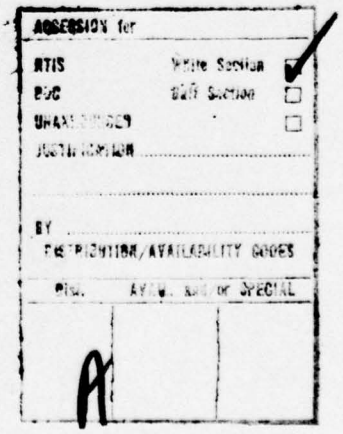
ABSTRACT

A characterization is given for the class of all closed loop eigenvector sets which can be obtained with a given set of distinct closed loop eigenvalues using state feedback. It is shown, furthermore, that the freedom one has in addition to specifying the closed loop eigenvalues is precisely this: to choose one set of closed loop eigenvectors from this class. Included in the proof of this result is an algorithm for computing the matrix of feedback gains which gives the chosen closed loop eigenvalues and eigenvectors. A design scheme based on these results is presented which gives the designer considerable freedom to choose the distribution of the modes among the output components. One interesting feature is that the distribution of a mode among the output components can be varied even if the mode is not controllable.



I. INTRODUCTION

One of the most popular techniques for altering the response characteristics of a control system is the application of linear state variable feedback. In the past decade, considerable effort has been made to understand exactly what feedback has to offer and what its limitations are.

The fact that one can use state feedback to assign the closed loop system any desired self conjugate set of eigenvalues, provided that the open loop system is controllable, is a well known and commonly used result. For single input systems, this result is simple to derive and has been known for some time. Eigenvalue placement in multi-input systems was studied by Lagenhop [1], Popov [2], Wonham [3], Simon and Mitter [4], and Brunovsky [5]. Wonham was the first to prove that this property of state feedback also applies to controllable multi-input systems.

Eigenvalue Assignment in multivariable systems is still not well understood, however. Unlike the single input case, specification of closed loop eigenvalues does not define a unique closed loop system. Many eigenvalue placement routines offer little freedom to exploit this nonuniqueness. Exceptions are algorithms which allow one to specify a number of components of the closed loop eigenvectors [6], [7], and algorithms designed to avoid large feedback gains ([8] chapter 6, [9]).

The purpose of this paper is to identify the freedom offered by state feedback beyond specification of the closed loop eigenvalues for the case in which the desired closed loop eigenvalues are distinct. In section 2 of the paper, a characterization is given for the class of all closed loop eigenvector sets which can be attained with a given set of distinct closed loop eigenvalues. It is shown, furthermore, that the freedom one has

beyond specifying the closed loop eigenvalues is precisely this: to choose one set from the class of "allowable" sets of closed loop eigenvectors. Included in section 2 is an algorithm for computing the feedback gain matrix which gives a closed loop system with the desired eigenvalues and eigenvectors. Section 3 illustrates one use of eigenvector assignment in control system design. Eigenvectors may be selected to adjust the distribution of the modes among the output components. One interesting feature is that the distribution of a mode among the output components can be varied even if the mode is not controllable.

Throughout the paper, the following notation will be used: R, C will denote, respectively, the field of real numbers and the field of complex numbers. For a matrix M which has n rows of numbers in R (C), Span {M} will be the subspace of R^n(C^n) spanned by the columns of M, while Ker {M} represents the Kernel of M (It should be clear from the context whether the operations are over R or C). If a property is said to hold for $i \in \underline{n}$, this means that it holds for all i satisfying $1 \leq i \leq n$. For a complex number c, c^* will represent its complex conjugate. A set is described as self conjugate if the complex conjugate of each of its members is contained in the set.

II. SIMULTANEOUS ASSIGNMENT OF CLOSED LOOP EIGENVALUES AND EIGENVECTORS

Consider the closed loop state equation

$$\dot{x}(t) = (A + BF)x(t)x(t)$$

obtained by applying linear state variable feedback $u(t) = Fx(t)$ to the system

$$\dot{x}(t) = Ax(t) + Bu(t)$$

where $x(t)\epsilon R^n$, $u(t)\epsilon R^m$, rank $B = m$. To aid in the development of the results, we associate with each number $\lambda\epsilon R$ (C) the matrix

$$S_\lambda = [\lambda I - A \; B]$$

and a compatibly partitioned matrix

$$K_\lambda = \begin{bmatrix} N_\lambda \\ M_\lambda \end{bmatrix}$$

whose columns constitute a basis for Ker $\{S_\lambda\}$. It is quite easy to show that the columns of N_λ are linearly independent if B has linearly independent columns, and that $N_{\lambda^*} = N_\lambda^*$.

It is well known that F may be chosen to yield any self conjugate set of closed loop eigenvalues provided that the (A, B) is controllable; that is, provided that

$$\text{rank } [B \quad AB \; . \; . \; . \; A^{n-1}B] = n$$

or, equivalently (see [8]), that rank $S_\lambda = n$ for all $\lambda\epsilon C$. Unless $m = 1$, however, specification of a set of closed loop eigenvalues does not uniquely define F. It is a simple matter to show that F <u>is</u> uniquely defined, if it exists, by the selection of a set distinct eigenvalues together with a corresponding set of eigenvectors. Hence the design freedom available beyond

eigenvalue selection is that of choosing one of the possible closed loop eigenvector sets.

Proposition 2.1, the main result of this paper, treats the case where the closed loop eigenvalues are distinct. For this case necessary and sufficient conditions for the existence of F which yields prescribed eigenvalues and eigenvectors are given.† The proof includes a procedure for computing F.

Proposition 2.1: Let $\{\lambda_i, i\epsilon\underline{n}\}$ be a self conjugate set of distinct complex numbers. There exists a matrix F of real numbers such that $(A + BF)\, v_i = \lambda_i v_i$ $(i\epsilon\underline{n})$ if and only if the following three conditions are satisfied for $i\epsilon\underline{n}$.

1. vectors v_i are linearly independent vectors in C^n
2. $v_i = v_j^*$ whenever $\lambda_i = \lambda_j^*$
3. $v_i \epsilon \text{Span}\{N_{\lambda_i}\}$.

If F exists and rank B = m, then F is unique.

Proof: (Sufficiency)

Suppose that v_i, $i\epsilon\underline{n}$, are chosen to satisfy the three conditions stated in the proposition. Since $v_i \epsilon \text{Span}\{N_{\lambda_i}\}$ for $i\epsilon\underline{n}$ (condition 3), then v_i can be expressed as $v_i = N_{\lambda_i} k_i$ for some vector $k_i \epsilon R^m(C^m)$, which implies that

$$(\lambda_i I - A)v_i + B M_{\lambda_i} k_i = 0.$$

If F is chosen so that $-M_{\lambda_i} k_i = Fv_i$, then $[\lambda_i I - (A+BF)]v_i = 0$. What remains in the proof is to show that a matrix F of real numbers satisfying

$$F[v_1\ v_2\ \ldots\ v_n] = [w_1\ w_2\ \ldots\ w_n] \qquad (2.1)$$

$$w_i = -M_{\lambda_i} k_i$$

† Although not presented in this form, this result was obtained independently and used implicitly by H. Kimura in his treatment of pole placement using output feedback [10].

can always be constructed.

If all n eigenvalues are real numbers, then v_i, w_i are vectors of real numbers and the matrix $[v_1\ v_2\ \ldots\ v_n]$ is nonsingular. For this case

$$F = [w_1\ w_2\ \ldots\ w_n][v_1\ v_2\ \ldots\ v_n]^{-1}.$$

For the case where there are complex eigenvalues, assume that $\lambda_1 = \lambda_2^*$. The second condition in the proposition states that $v_1 = v_2^*$ which implies that $w_1 = w_2^*$. The equation which must be solved then is

$$F[v_{1R}+jv_{1I}\ \ v_{1R}-jv_{1I}\ V] = [w_{1R}+jw_{1I}\ w_{1R}-jw_{1I}\ W] \quad (2.2)$$

where the columns of V and W are v_i, i=3, ...,n, and w_i, i=3, ...,n, respectively. Multiplication of both sides of equation (2.2) from the right by the nonsingular matrix

$$\begin{bmatrix} \frac{1}{2} & -j\frac{1}{2} & \\ & & 0 \\ \frac{1}{2} & +j\frac{1}{2} & \\ - & - - - - - & \\ 0 & & I \end{bmatrix}$$

yields the equivalent equation

$$F[v_{1R}\ v_{1I}\ V] = [w_{1R}\ w_{1I}\ W].$$

Clearly since v_i, i=1,n, are independent, the columns of $[v_{1R}\ v_{1I}\ V]$ are linearly independent. This procedure can obviously be applied for all complex pairs of eigenvalues.

(Necessity)

Necessity of the first two conditions follows directly from elementary matrix theory. Furthermore if $(A+BF)v_i = \lambda_i v_i$ then $[\lambda_i I - A]v_i = BFv_i$; written differently,

$$[\lambda_i I - A \;\; B]\begin{bmatrix} v_i \\ -Fv_i \end{bmatrix} = 0. \qquad (2.3)$$

Since the columns of K_{λ_i} form a basis for the kernel of $[\lambda_i I - A \;\; B]$, it follows that $v_i \in \text{Span}\{N_{\lambda_i}\}$.

Since A+BF is uniquely defined by its (distinct) eigenvalues and eigenvectors, it is clear that F is unique *whenever* B *has independent* column vectors.

Q.E.D.

Remark: For the case where the selected eigenvalues are not distinct, the conditions of Proposition are sufficient, and the last two conditions are necessary.

It is interesting that controllability is not mentioned in this proposition. The equivalence of controllability and pole assignability implies that the three conditions in Proposition 2.1 cannot be satisfied if the uncontrollable eigenvalues are not included in the selected set of closed loop eigenvalues. This point deserves a few words of clarification.

If (A, B) is not controllable, then there exists an open loop eigenvalue λ_i and a vector p_i satisfying $p_i^T S_{\lambda_i} = 0$. This implies that p_i is an eigenvector of A^T which is invariant under feedback; i.e.

$$(A+BF)^T p_i = A^T p_i = \lambda_i p_i .$$

Now for any λ and $v \in \text{Span}\{N_\lambda\}$ there exists a matrix F (see the proof of Proposition 2.1) satisfying $(A+BF)v = \lambda v$. Hence

$$\lambda_i (p_i^T v) = p_i^T (A+BF) v = \lambda (p_i^T v)$$

and it follows that $p_i^T N_\lambda = 0$ for all $\lambda \neq \lambda_i$.

The important point is this: if the eigenvalues of the uncontrollable subsystem obtained using Kalman's controllability decomposition (see [11] or almost any linear systems text) are distinct, then they may be included in the set of selected closed loop eigenvalues without violating the distinct eigenvalue assumption of Proposition 2.1. Even though an uncontrollable eigenvalue λ_i is invariant under feedback, there is considerable freedom, as will be shown in section 3, to select $v_i \in \text{Span}\{N_{\lambda_i}\}$, although it is clear from the last paragraph that v_i must satisfy $p_i^T v_i \neq 0$.

III. APPLICATION TO CONTROL SYSTEM DESIGN

While the overall speed of response of the closed loop system is determined by its eigenvalues, the "shape" of the transient response depends to a large extent on the closed loop eigenvectors. In the following paragraphs it is shown that through eigenvector selection the designer has considerable freedom to adjust the distribution of the modes among the various output components. In this sense, he may "shape" the response characteristics of the system.

Consider again the closed loop system

$$\dot{x}(t) = (A+BF)x(t)$$

together with an output equation

$$y(t) = Cx(t)$$

where $y(t) \epsilon R^r$, rank B=m, rank C=r, $m \geq r$. In terms of the (distinct) closed loop eigenvalues λ_i, $i \epsilon \underline{n}$, and eigenvectors v_i, $i \epsilon \underline{n}$, the output vector is given by

$$y(t) = \sum_{i=1}^{n} Cv_i (p_i^T x_o) e^{\lambda_i t}$$

where

$$[p_1\ p_2 \cdots p_n]^T = [v_1\ v_2 \cdots v_n]^{-1}.$$

In other words, $y(t) = \sum_{i=1}^{n} \alpha_i e^{\lambda_i t}$ where α_i is proportioned to Cv_i. Hence if $Cv_i = [1\ \ 0 \cdots 0]^T$ then the ith mode appears only in the first output component. If $Cv_i = [2\ \ 1\ \ 0 \cdots 0]^T$, it appears in the first two outputs and is twice as large in $y_1(t)$ as it is in $y_2(t)$, etc.

Now consider the freedom available to assign Cv with, $v \epsilon N_\lambda$, where λ is an arbitrary number in R (C). The columns of CN_λ span $R^r(C^r)$, meaning

that cv can be arbitrarily assigned, if and only if

$$\text{rank} \begin{bmatrix} \lambda I - A & B \\ C & 0 \end{bmatrix} = r + \text{rank } [\lambda I - A \quad B] \tag{3.1}$$

To see this, observe that there exists $v \in \text{Span } N_\lambda$ such that $Cv = e$ if and only if there is a solution to the equation

$$\begin{bmatrix} \lambda I - A & B \\ C & 0 \end{bmatrix} \begin{bmatrix} v \\ w \end{bmatrix} = \begin{bmatrix} 0 \\ e \end{bmatrix} \tag{3.2}$$

A solution to this equation exists for every vector e iff

$$\text{rank} \begin{bmatrix} \lambda I - A & B \\ C & 0 \end{bmatrix} = \text{rank} \begin{bmatrix} \lambda I - A & B & 0 \\ C & 0 & I \end{bmatrix} .$$

The rightmost matrix has rank $r + \text{rank}[\lambda I - A \quad B]$. If (C, A) is observable, this can be stated equivalently as follows: Cv, $v \in \text{Span } N_\lambda$, can be arbitrarily assigned provided that λ is not a zero (as defined by Rosenbrock [12]) of the transfer function matrix.

Even with the freedom to assign Cv, $v \in N_\lambda$, which exists if equation 3.1 holds, complete freedom to assign Cv_i, $i \in \underline{n}$, where v_i are closed loop eigenvectors, does not exist: the vectors v_i, $i \in \underline{n}$, must satisfy the conditions of proposition 2.1. There are a few obvious constraints on the vectors Cv_i, $i \in \underline{n}$. It is clear, for example, that if m=r, λ is not an uncontrollable eigenvalue, and (3.1) holds, then $Cv_i = 0 \Rightarrow v_i = 0$ which is not valid. Considering the linear independence requirement, it is also clear that the number of modes which can be restricted to a single output component is less than or equal to n-r+1.

The case in which an eigenvalue λ_i is uncontrollable is quite interesting. In the last section it was shown that the closed loop eigenvector v_i must satisfy $p_i^T v_i \neq 0$, where $A^T p_i = \lambda_i p_i$. If

$$\text{rank}\begin{bmatrix}\lambda I-A & B & 0\\ C & 0 & 0\\ p_i^T & 0 & 1\end{bmatrix} = \text{rank}\begin{bmatrix}\lambda I-A & B\\ C & 0\\ p_i^T & 0\end{bmatrix} \qquad (3.3)$$

then there exists a vector v_i, $v_i \in N_{\lambda_i}$, such that $Cv_i = 0$, $p_i^T v_i \neq 0$. This implies that the mode can be eliminated entirely from the output response (i.e. made unobservable) provided that the remaining vectors v_j, $j \in \underline{n}$, $j \neq i$, can be chosen to satisfy the conditions of Proposition 2.1.

To complete this section, we illustrate one design scheme which allows a degree of transient response shaping. It is assumed that the uncontrollable eigenvalues are distinct. The steps of the procedure are as follows:

1. Select distinct closed loop eigenvalues -- the set must include uncontrollable eigenvalues.
2. Select e_i, the desired value of cv_i, for each eigenvalue. This choice should be made on the basis of the desired distribution of modes among output components.
3. Compute v_i, w_i satisfying

$$\begin{bmatrix}\lambda_i I-A & B\\ C & 0\end{bmatrix}\begin{bmatrix}v_i\\ w_i\end{bmatrix} = \begin{bmatrix}0\\ e_i\end{bmatrix}.$$

If λ_i is an uncontrollable eigenvalue, the solution must satisfy $p_i^T v_i \neq 0$. This inequality may be forced by solving

$$\begin{bmatrix}\lambda_i I-A & B\\ C & 0\\ p_i^T & 0\end{bmatrix}\begin{bmatrix}v_i\\ w_i\end{bmatrix} = \begin{bmatrix}0\\ e_i\\ k\end{bmatrix}, \quad k \neq 0$$

if the leftmost matrix has rank $r+1+\text{rank}[\lambda_i I-A \;\; B]$.

4. If the vectors v_i, $i \in \underline{n}$ are not linearly independent, alter one or more of the vectors e_i, $i \in \underline{n}$, and return to step 3; otherwise proceed to step 5.
5. Compute the feedback matrix and the closed loop system matrix. If the transient response characteristics are not satisfactory alter one or more of the vectors e_i, according to the nature of the response, and return to step 3.

Example: Consider the open loop system with matrices

$$A = \begin{bmatrix} -1.25 & 0.75 & -0.75 \\ 1 & -1.5 & -0.75 \\ 1 & -1 & -1.25 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 1 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

which has controllable eigenvalues at -1.25, -2.25, and an uncontrollable eigenvalue at -0.5.

To illustrate the response shaping feature of the procedure suggested in this section, it is assumed that the objective is to shift the controllable eigenvalues to -5., -6., and to obtain a "reasonable" response for the initial condition $[0 \quad 0 \quad 1]^T$, which represents a disturbance in $x_3(t)$. Table 1* gives three different closed loop systems which have eigenvalues $\lambda_1 = -.5$, $\lambda_2 = -5$, $\lambda_3 = -6$, but which differ in their eigenvectors.

The first system in Table 1 was obtained using a standard modal controller (see [8], chapters 5,6) of the form

---

* Computed answers have been rounded to three significant figures. Numbers smaller in magnitude than 10^{-15} are shown as zero.

$$u(t) = (\alpha_1 g_1 p_1^T + \alpha_2 g_2 p_2^T)x(t)$$

where the following equations hold:

$$A^T p_1 = -1.25 p_1, \quad p_1^T p_1 = 1$$

$$(A + B g_1 p_1^T) p_2 = -2.25 p_2, \quad p_2^T p_2 = 1 .$$

The constants α_1, α_2 and the vectors g_1, $g_2 \in R^n$ were chosen to shift one eigenvalue from -1.25 to -5.0, and then the second eigenvalue from -2.25 to -6.0, and to minimize the gain required for each shift. Figure 1 shows the response of this system to a unit disturbance in $x_3(t)$.

The response of the first system is not satisfactory because of the rather large slow mode component in $y_1(t)$. This mode is uncontrollable, satisfies equation (3.3), and is eliminated in the output of system 2 (Figure 2). In this system e_2, e_3 were chosen to be equal to cv_2^1, cv_3^1, where v_2^1, v_3^1 are the eigenvectors corresponding to λ_2, λ_3 in system 1. Note that the entries of cv_2^1 have opposite signs.

To further illustrate the spirit of the procedure, it is assumed that a positive going transient in $y_1(t)$ is desired for a positive disturbance in a $x_3(t)$. This is accomplished in system 3 by simply choosing e_2 to have components of the same sign, in this case $e_2 = [.5 \quad .5]^T$. Figure 3 gives the response of the third system.

IV. CONCLUSIONS

Necessary and sufficient conditions for simultaneous eigenvalue, eigenvector assignment have been given for the case where the desired eigenvalues are distinct. The corresponding design procedure based on this result gives the designer considerable freedom to select the distribution of modes (whether controllable or not) among the output components.

The shortcoming of the design procedure is that it is *ad hoc* in nature. In future research, an attempt will be made to develop a systematic design procedure allowing the designer to "shape" in some sense the average transient response characteristics of the system.

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	Selected Vectors e_1 e_2 e_3	Closed Loop Matrix A+BF
System 1	modal control used	-5.78 5.28 -7.44 -.0784 -.421 -4.79 -.0784 .0784 -5.29
System 2	0 -.889 .990 0 .323 .0990	-5.78 0 -2.16 -.0784 -.5 -4.72 -.0784 0 -5.22
System 3	0 .5 .990 0 .5 -.0990	-6.11 0 1.11 -.111 -.5 -4.39 -.111 0 -4.89

Table 1

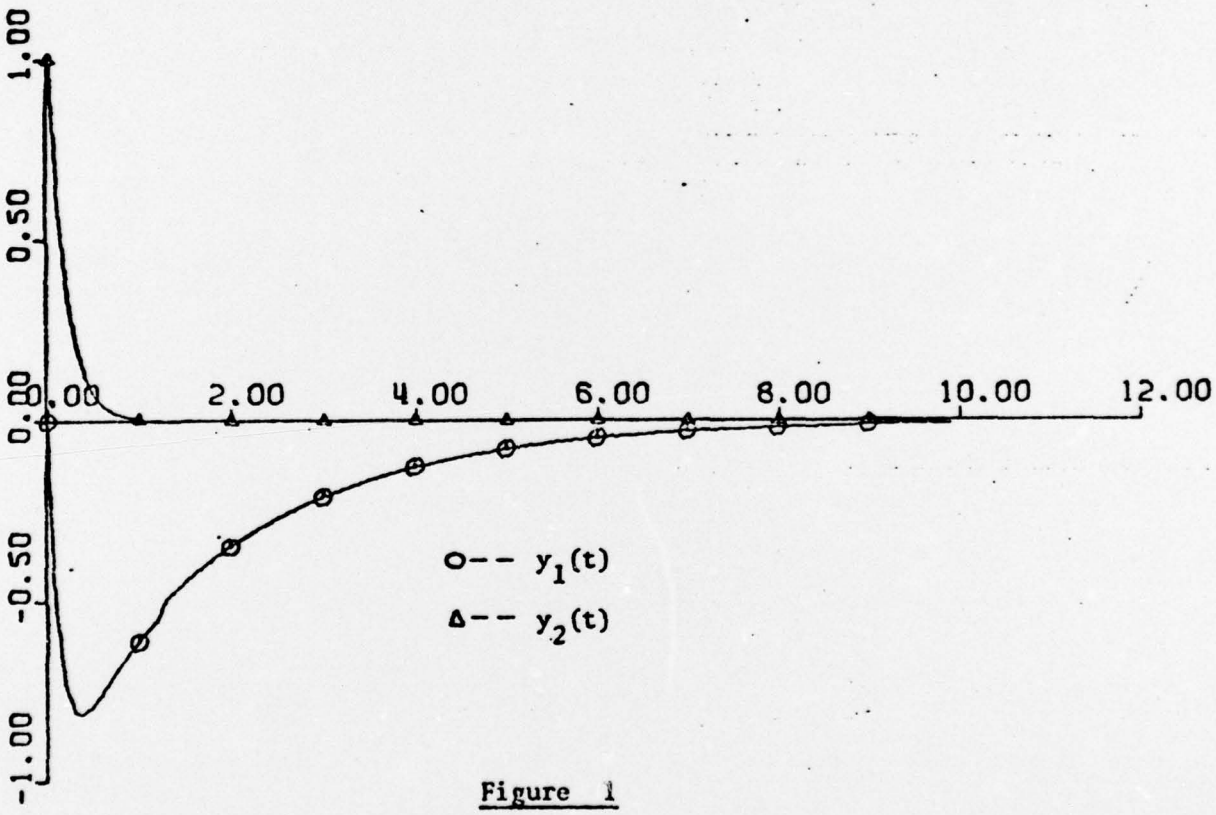

Figure 1

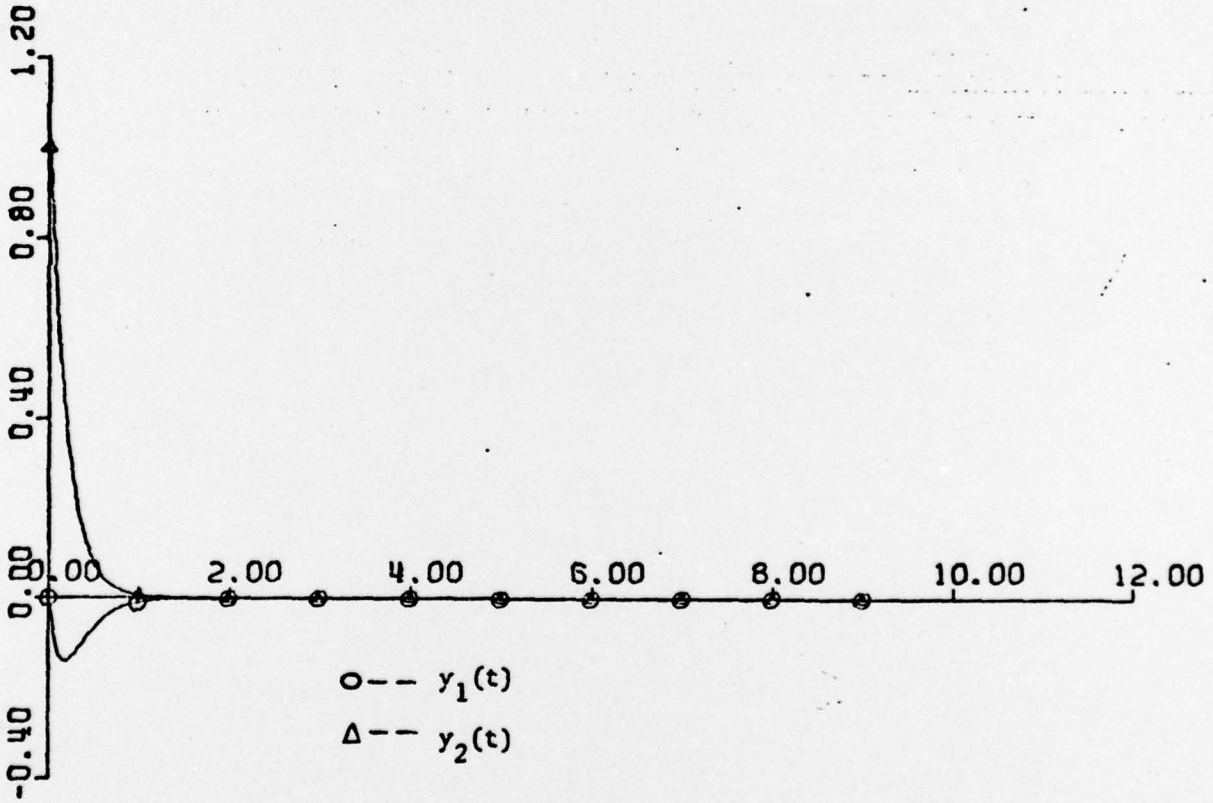

Figure 2

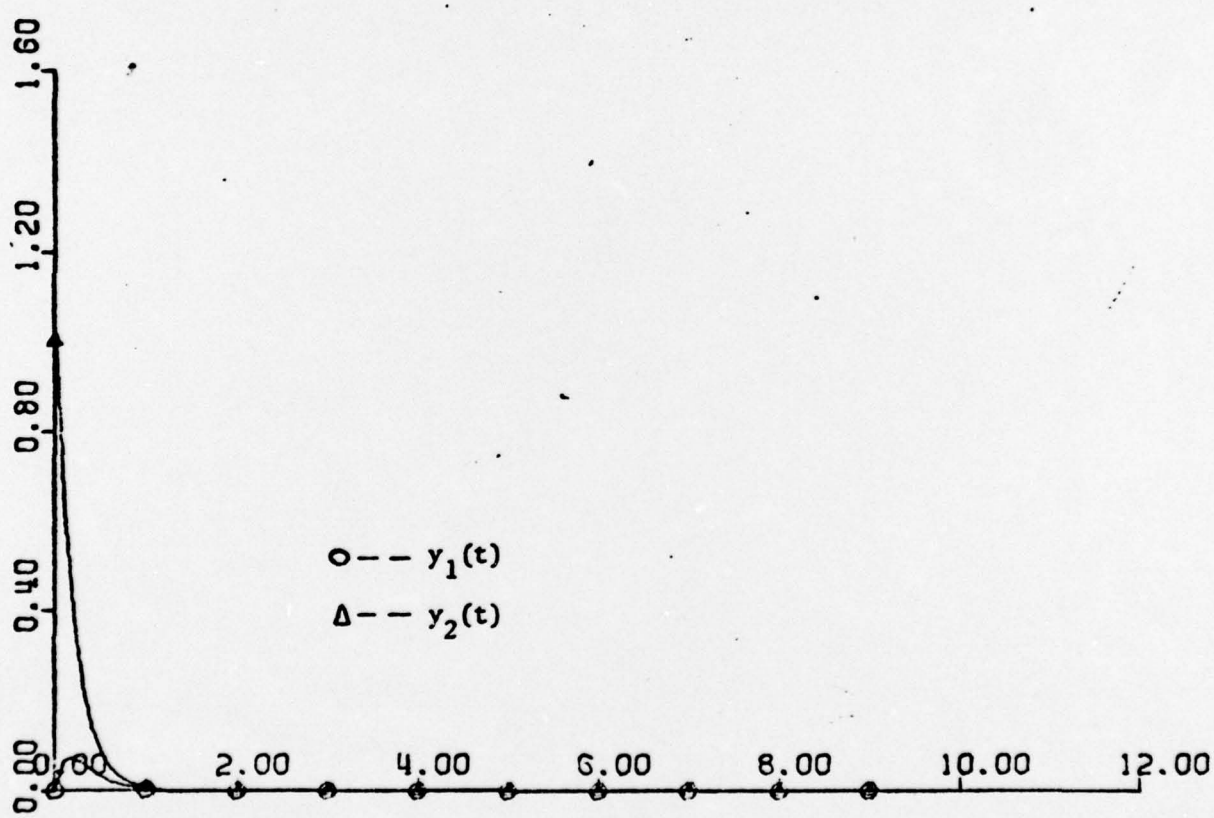

Figure 3

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A characterization is given for the class of all closed loop eigenvector sets which can be obtained with a given set of distinct closed loop eigenvalues using state feedback. It is shown, furthermore, that the freedom one has in addition to specifying the closed loop eigenvalues is precisely this: to choose one set of closed loop eigenvectors from this class. Included in the proof of this result is an algorithm for computing the matrix of feedback gains which gives the chosen closed loop eigenvalues and eigenvectors. A design scheme based on these results is presented which gives the designer considerable freedom to choose the →

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20 Abstract

distribution of the modes among the output components. One interesting feature is that the distribution of a mode among the output components can be varied even if the mode is not controllable.